Unit-1

Introduction

Charge: The basic quantity in an electric circuit is the electric charge.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

Addition or deficiency of electrons causes charge (q)

The charge on an electron is negative and equal in magnitude to 1.602×10^{-19} C

In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons.

Solved Problem: What quantity of charge is carried by 6.24×10^{21} electrons?

The charge on an electron is 1.602×10^{-19} Coulombs

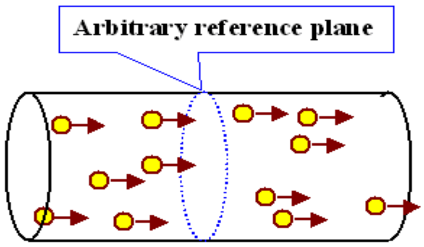
$$1e^{-} = 1.602 \times 10^{-19} \; C$$

$$1C = \frac{1}{1.602 \times 10^{-19}} \; electrons$$

$$= 6.24 \times 10^{18} \; electrons$$

$$\frac{6.24 \times 10^{21}}{6.24 \times 10^{18}} = 10^{3} = 1000\,C$$

Electric Current: The flow of electric charges.



Electric Current is the time rate of change of charge, measured in ampere (A).

$$I = \frac{Q}{t} \quad or \quad i = \frac{dq}{dt}$$

1 ampere= 1 coulomb/ second

Current must be designated with both a direction and a magnitude

Solved Problem: If a current of 5A flows for 2 minutes, find the quantity of charge transferred.

Solution: Quantity of electricity Q= It coulombs

I =5A

t =2×60=120 sec

Hence Q=5×120=600 C

Voltage: To move the electron in a conductor in a particular direction requires some work or energy transfer performed by an external electro motive force (emf). Voltage also known as voltage or potential difference.

Voltage (or potential difference) is the energy required to move a unit charge through an element , measured in volts (V).

The voltage between two point a and b in electric circuit is the energy (work) needed to move 1 C of charge from a to b :

$$v_{ab} = \frac{dw}{dq} \quad or \quad V = \frac{W}{Q}$$

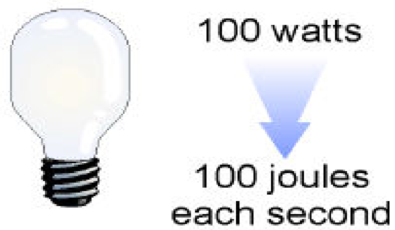
Where w =energy (J), q = charge (C)

1 volt = 1 joule/coulomb = 1 newton meter/ coulomb

Power and Energy : Power is the time rate of expending or absorbing energy, measured in watts . It is the rate at which energy is used.

Energy is the capacity to do work (to push electrons through a material)

$$p = \frac{dw}{dt} \quad p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v\,i \quad P = V\,I$$



Where w =energy (J), t = time (s)

Since energy is measured in joules, power is measured in joules per second.

One joule per second is equal to one watt.

$$P = \frac{W}{t} \qquad W = Pt$$

Solved Problem: A source e.m.f. of 5V supplies a current of 3A for 10 minutes. How much energy is provided in this time?

Solution: Energy = Power × time and Power = Voltage × Current.

Hence Energy = V I t

= 5×3×(10×60) = 9000Ws or J = 9kJ

Solved Problem: An electric heater consumes 1.8 MJ when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Solution: Energy = Power × time , hence

Power = Energy/time

= (1.8 × 10^6) J/(30 × 60) s

=1000 J/s=1000W

i.e. Power rating of heater=1kW

Power P=VI, thus I = P/V = 1000/250 = 4A

Hence the current taken from the supply is 4A

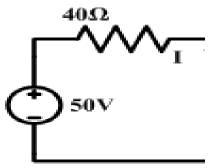
Concepts of Circuit/Network

An electric network : is defined as an interconnection of two or more electrical elements.

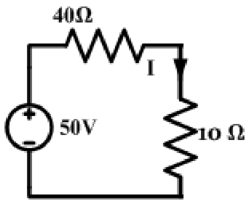
Circuit :A network that contains at least one closed path is known as circuit

All circuits can be called as networks

But all networks are not circuits

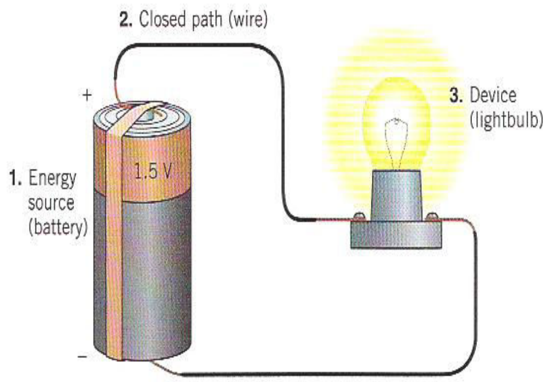


Network



Circuit

All electric circuits have three main parts



1. A source of energy
2. A closed path
3. A device which uses the energy

If Any part of the circuit is open the device will not work!

Classification of Network Elements

- Active & Passive elements
- Bilateral & Unilateral elements
- Linear & Non linear elements

Active & Passive elements

Passive Elements: The elements that absorbs or stores energy is called passive element. These are not capable of generating energy

Examples: Resistor (R), Capacitor (C) , Inductor (L) , Transformer

Active Elements: The elements that supply energy to the circuit is called active element.

These are capable of generating energy

Examples: Voltage and Current sources, Generators,

Bilateral & Unilateral elements

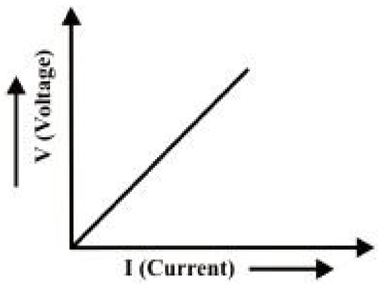
Bilateral Elements: Conduction of current in both directions in an element with same magnitude is termed as bilateral element.

Examples: Resistance; Inductance; Capacitance

Unilateral Elements: Conduction of current in one direction in an element is termed as unilateral element

Examples: Diode, Transistor.

Linear & Non linear elements

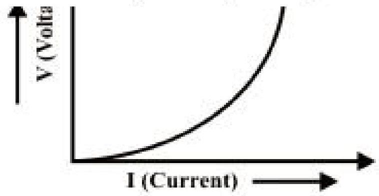


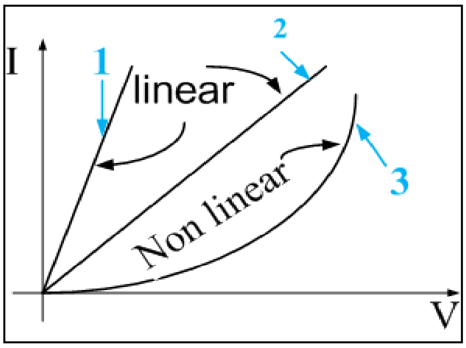
Linear Element: The elements that obeys ohm's law and homogeneity principle is called Linear element.

Examples: Resistor (R), Capacitor (C), Inductor (L)

Non-Linear Element: The elements that does not obey ohm's law and homogeneity principle is called Non-Linear element.

Examples: Semiconductors, Diode, Transistor





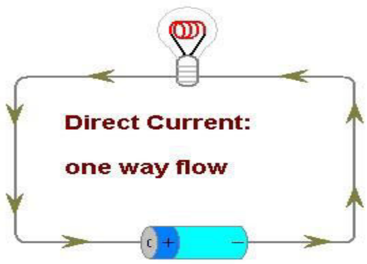
Linear elements are the elements that shows a linear relation ship between voltage and current as shown in curve 1 & 2

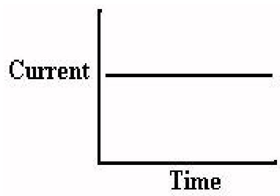
Non linear elements are the elements that doesnt show a linear relation ship between voltage and current as shown in curve 3

Direct and Alternating Current (DC &AC)

DC = Direct Current - current flows in one direction

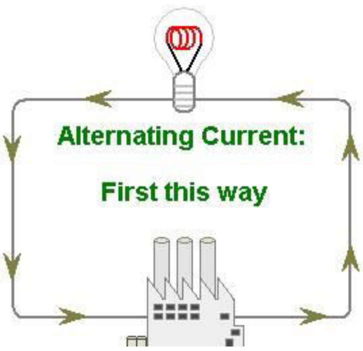
Example: Battery

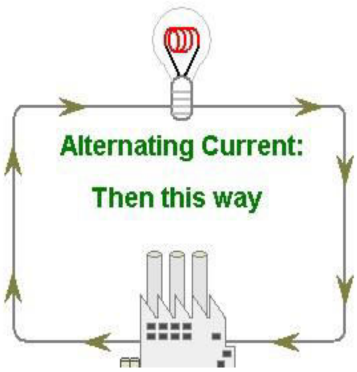


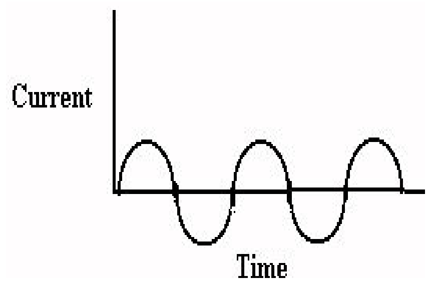


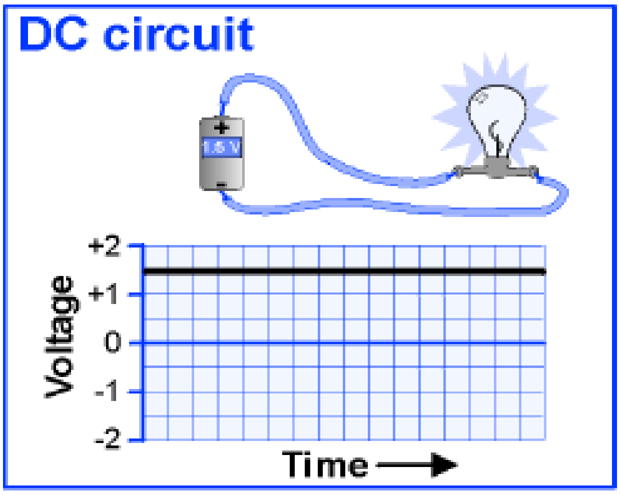
AC = Alternating Current- current reverses direction many times per second.

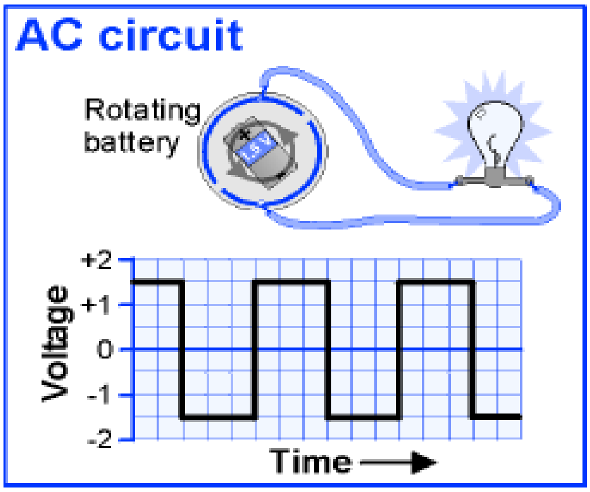
Example: Wall outlet (progress energy)









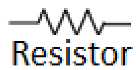


Passive Elements:

The passive elements are three

1. Resistance 2. Inductance 3. Capacitance

1. Resistance:



Definition: The tendency for a material to oppose the flow of electrons

Due to resistances electrical energy changes into thermal energy and light

Example: light bulb filament

Resistance is measured in Ohms (Ω)

Resistor : An object that has a given resistance is known as resistor

Ohm's law:

It defines the relationship between voltage, current, and resistance in an electric circuit

Electric Current in a conductor is direct proportional to the voltage applied to it at constant temeperature.

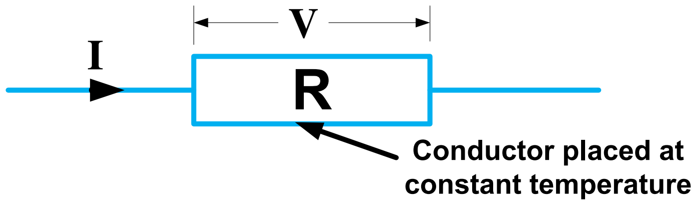
The proportionality constant is inverse of resistance of conductor

According to definition

$$I \,\alpha\, V$$

$$I = \frac{1}{R} V$$

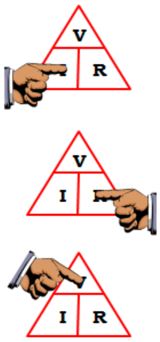
$$V = I\,R$$



Where, V = Voltage across the conductor in volts

I = Current flowing through the conductor in Ampere

R = Proportionality constant (resistance in ohms)



$$I = \frac{V}{R} \quad (\text{amperes}, A)$$

$$R = \frac{V}{I} \quad (\text{ohms}, \Omega)$$

$$V = IR \quad (\text{volts}, V)$$

The resistance (R) is defined as the ratio of the voltage V applied across a piece of material to the current I through the material.

Factors affecting the Resistance:

1. Length of the material:

The Resistance "R" is directly proportional with its length: **" *l* "** $R \propto l$

As length of the wire increases, resistance also increases.

2. Cross Sectional area of the material:

The Resistance "R" is inversely proportional with its Cross Sectional Area: **" *a* "**

$$R \propto \frac{1}{a}$$

As Cross Sectional Area of the wire increases, resistance also decreases.

3. Nature of the material:

The Resistance "R" is dependent on the Nature of the material

i.e specific resistivity or specific resistance ρ

- In Conductors, No of free electrons are very high so resistance of the conductor is very less.
- In Insulators and Semi conductors, No of free electrons are less so resistance of the conductor is very high.

4. Temperature of the conductor:

The Resistance "R" is dependent on the Temperature of the conductor.

$$R_2 = R_2(1+\alpha\ \Delta t)$$

Where 'α' is the temperature coefficient of resistance

- For Conductors 'α' = **+ve, as** temperature increases, resistance also increases.
- For Insulators and Semi conductors 'α' = **-ve, as** temperature increases, resistance decreases.

So neglecting the effect of temperature we can write

$$\mathbf{R \propto l \Rightarrow R \propto \frac{1}{a} \Rightarrow R = \rho \frac{l}{a}}$$

Where l = length of the conductor,
a = area of cross section, and
ρ = specific resistance or resistivity of the material this factor will change for each material

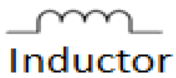
Thus, when the current is carried by the resistor,

1. The voltage is dropped across it and is given by V=IR
2. Also the heat dissipation occurs $P = I^2R$.

Therefore, the resistance is an heat dissipation element and never stores the energy

Inductance:

Definition: The flux linkages per ampere current.



$$L = \frac{N\varphi}{i} \text{ where } L = \text{inductance in Henry, } N\phi = \text{Flux linkages in Wb - T, } i = \text{current in Ampere}$$

When the inductor is carrying an alternating current with di/dt, then an voltage is induced across it. The magnitude of the voltage induced is directly proportional to the rate of change of current

$$\text{i.e, } V \alpha \frac{di}{dt} \Rightarrow \text{therefore, the voltage is } V = L\frac{di}{dt}$$

The current in the inductor is obtained from the above defined voltage equation

$$\frac{di}{dt} = \frac{V}{L} \Rightarrow di = \frac{V}{L}dt \Rightarrow \int_0^t di = \frac{1}{L}\int_0^t Vdt \Rightarrow i(t) = \frac{1}{L}\int_0^t Vdt + i(0)$$

The power consumed by the inductor is $p = vi = Li\frac{di}{dt}$

The energy stored in the inductor is $W = \int_0^t pdt = \int_0^t Li\frac{di}{dt}dt = \int_0^t Lidi = \frac{1}{2}Li^2$

Salient points:

1. Inductor behaves as a short circuit element for dc supply

2. Inductor doesn't allows the sudden changes in the current

3. Inductor stores the energy even if the voltage across it is zero and stores the energy in the form of magnetic field

4. Inductor is an energy storing element and never dissipates the energy, whereas the practical inductor dissipates the energy due to its internal resistance.

<u>Capacitance:</u>

Definition: The Charge per unit Voltage.

$C = \frac{Q}{V}$ where C = capacitance in Farad, Q = Charge in Coloumb, V = voltage in Volt

When the capacitor is applied an alternating voltage with dv/dt, then the current in it is given by

$$i = \frac{dq}{dt} \Rightarrow i = C\frac{dv}{dt}$$

The voltage across the capacitor is obtained from the above defined current equation

$$\frac{dv}{dt} = \frac{i}{C} \Rightarrow dv = \frac{i}{C}dt \Rightarrow \int_0^t dv = \frac{1}{C}\int_0^t idt \Rightarrow v(t) = \frac{1}{C}\int_0^t idt + v(0)$$

The power consumed by the capacitor is $p = vi = Cv\frac{dv}{dt}$

The energy stored in the capacitor is $W = \int_0^t pdt = \int_0^t Cv\frac{dv}{dt}dt = \int_0^t Cvdv = \frac{1}{2}Cv^2$

Salient points:

1. Capacitor behaves as a open circuit element for dc supply

2. Capacitor doesn't allows the sudden changes in the voltage

3. Capacitor stores the energy even if the current in it is zero and stores the energy in the form of electrostatic field

4. Capacitor is an energy storing element and never dissipates the energy, whereas the practical capacitor dissipates the energy due to its internal resistance.

Solved Problem: A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm². The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ-m. Find the resistance of the coil

Solution: Length of the coil, $l = 0.8 \times 2000 = 1600$ m

Area of Cross section $a = 0.8\ mm^2 = 0.8 \times 10^{-6} m^2$

Resistivity $\rho = 0.02\ \mu\Omega$-m

$$R = \rho \frac{l}{a} = 0.02 \times 10^{-6} \times \frac{1600}{0.8} = 40\ \Omega$$

Solved Problem: A rectangular carbon block has dimensions 1.0cm x1.0cm x 50 cm. Resistivity of carbon at 20°C is $3.5 \times 10^{-5}\ \Omega$-m.

(i) What is the resistance measured between the two square ends?

(ii) What is the resistance measured between two opposing rectangular faces

Solution:

(i) Length of the block, ***l* =0.5 m** Area of Cross section ***a* = 1x1 = 1 cm² =10^{-4}m²**

Resistivity $\rho = 3.5 \times 10^{-5}\ \Omega$-m

$$R = \rho \frac{l}{a} = 3.5 \times 10^{-5} \times \frac{0.5}{10^{-4}} = 0.175\ \Omega$$

(ii) Length of the block, ***l* =1 cm** Area of Cross section ***a* = 1x50 = 50cm² =5×10^{-3}m²**

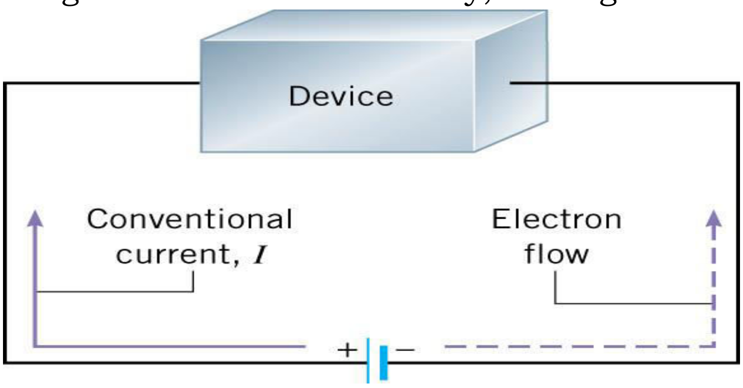
Resistivity $\rho = 3.5 \times 10^{-5}\ \Omega$-m

$$R = \rho \frac{l}{a} = 3.5 \times 10^{-5} \times \frac{10^{-2}}{5 \times 10^{-3}} = 7 \times 10^{-5}\ \Omega$$

Current Flow

The electrons flow out of the negative side of the battery, through the circuit, and back to the positive side of the battery.

Conventional Current assumes that current flows out of the positive side of the battery, through the circuit, and back to the negative side of the battery.



Conventional current is the hypothetical flow of positive charges that would have the same effect in the circuit as the movement of negative charges that actually does occur.

Solved Problem: The flashlight uses two 1.5 V batteries to provide a current of 0.4 A in the filament. Determine the resistance of the glowing filament.

Solution: $$R = \frac{V}{I} = \frac{3.0\text{ V}}{0.4\text{ A}} = 7.5\,\Omega$$

Solved Problem: An electric iron draws 2 A at 120 V. Find its resistance.

Solution: From Ohm's law, Resistance $R = \frac{V}{I} = \frac{120\text{ V}}{2\text{A}} = 60\,\Omega$

Solved Problem: The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ohms at 110 V?

Solution: From Ohm's law, Current $I = \frac{V}{R} = \frac{110\text{ V}}{15\,\Omega} = 7.333\text{ A}$

Solved Problem: Determine the p.d. which must be applied to a 2k resistor in order that a current of 10mA may flow.

Solution:

$$R = 2\ k\Omega = 2\times10^{3}\ \Omega$$

$$I = 10\ mA = 10\times10^{-3}\ A = 0.01\,A$$

From Ohm's law, $V = R\,I = (0.01)(2000) = 20V$

Solved Problem: A 100V battery is connected across a resistor and causes a current of 5mA to flow. Determine the resistance of the resistor. If the voltage is now reduced to 25V, what will be the new value of the current flowing?

Solution:

Resistance $R = \frac{V}{I} = \frac{100}{5\times10^{-3}} = 20\,k\Omega$

Current when voltage is reduced to 25V, $I = \frac{V}{R} = \frac{25}{20\times10^{3}} = 1.25\,mA$

Other useful formulas :

$$V = R\,I \quad \& \quad P = V\,I$$

$$Power\ \ P = (R\,I)\,I = I^{2}R$$

$$I = \frac{V}{R} \quad \& \quad P = V\,I$$

$$Power\quad P = V\left(\frac{V}{R}\right) = \frac{V^{2}}{R}$$

Solved Problem: Calculate the power dissipated when a current of 4mA flows through a resistance of 5 k

Solution: Power $P = I^2R = (4\times10{-}3)^2(5\times10^3)$

$=16\times10^{-6}\times5\times10^3 = 80\times10^{-3}$

$= 0.08W$ or $80mW$

Alternatively, since $I = 4\times10^{-3}$ and $R = 5\times10^3$

then from Ohm's law, voltage $V = I R = 4\times10^{-3}\times5\times10^{-3} = 20V$

Hence, power $P = V\times I = 20\times4\times10^{-3} = 80mW$

Solved Problem: An electric heater consumes 3.6 MJ when connected to a 250V supply for 40 minutes. Find the power rating of the heater and the current taken from the supply.

Solution

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{3.6\times10^6}{40\times60}\frac{\text{J}}{\text{s}}\ (\text{or W}) = 1500\,\text{W}$$

i.e. Power rating of heater $=$ **1.5 kW**

$$\text{Power } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1500}{250} = 6\,\text{A}$$

Hence the current taken from the supply is **6 A**

Active Elements:

The most important active elements are voltage or current sources that generally deliver power to the circuit.

Active elements (Sources) are classified as

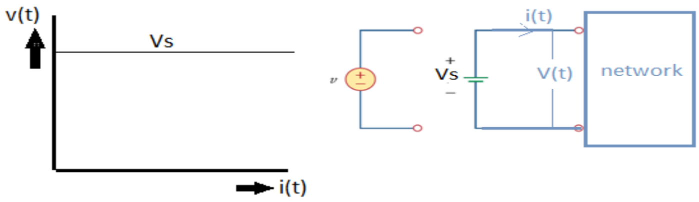
- Independent Sources
- Dependent Sources

Independent source: is an active element that provides a specified voltage or current that is completely independent of other circuit elements

Ideal Voltage source:

The ideal voltage source delivers V_s as v(t) for all the values of i(t)

Therefore, $v(t) = V_s$ for all values of $i(t)$ and its V-I characteristics are shown in below fig.



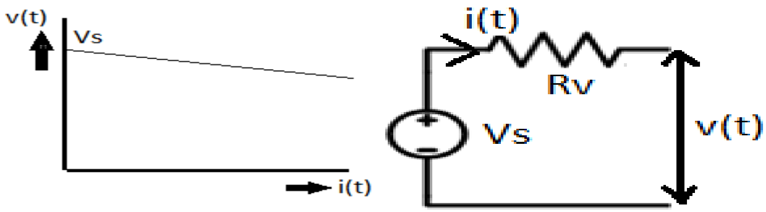
Ideal Voltage Source and V-I Characteristics

Practical Voltage source:

The Practical voltage source delivers v(t) and i(t) from the generated V_s against its internal resistance R_v

$v(t) = V_s$ for $i(t) = 0$

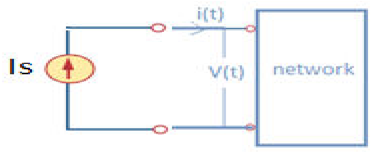
$v(t) = V_s - i(t)R_v$ for $i(t) > 0$



Practical Voltage Source and its V-I Characteristics

Ideal Current source:

The ideal current source delivers Is as i(t) for all the values of v(t) Therefore, $i(t) = I_s$ for all values of v(t) and its V-I characteristics are shown in the adjacent figure



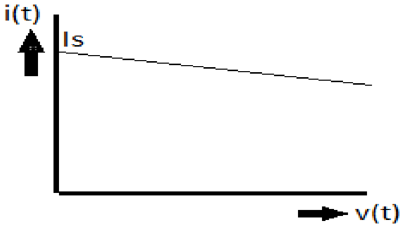
Ideal Current Source and V-I Characteristics

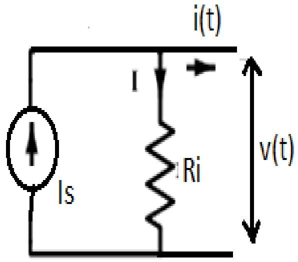
Practical Current source:

The Practical current source delivers i(t) and v(t) from the generated I_s against its internal resistance R_i

$i(t) = I_s$ for $v(t) = 0$ and

$$i(t) = I_s - \frac{v(t)}{R_i} \text{ for } v(t) > 0$$



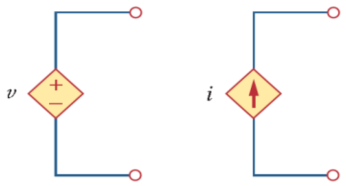


Practical Current Source and V-I Characteristics

- ***The internal resistance of an ideal voltage source is zero***
- ***The internal resistance of an ideal current source is infinity***

Dependent (controlled) source : is an active element that provides a specified voltage or current controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols

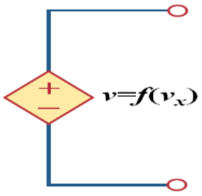


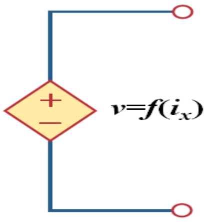
Dependent voltage Source ***Dependent Current source***

The control of the dependent source is achieved by a voltage or current of some other element in the circuit

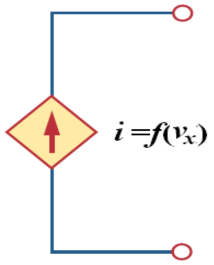
There are four possible types:

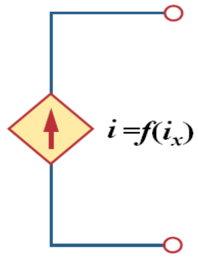
1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)





Voltage-controlled voltage source current-controlled voltage source





Voltage-controlled current source   current-controlled current source

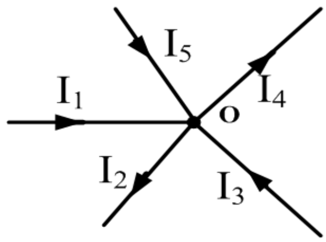
Kirchhoff Laws:

Gustav Kirchhoff (1824-1887), an eminent Germen physicist did a considerable amount of work on the principle of governing behavior of electric circuits. He gave his finding in a set of two laws which together called Kirchhoff's laws. These two laws are

1. Kirchhoff's Current Law (KCL) 2. Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law states that the algebraic sum of the current meeting at a node (junction) is equal to zero



i.e., $\sum I = 0$

This law is illustrated below.

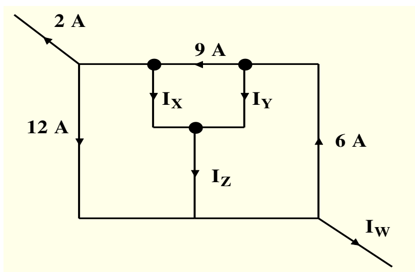
Five branches are connected to node O which carries currents I_1, I_2, I_3, I_4 and I_5 as shown in figure (1). Consider current entering (I_1, I_3 & I_5) to the node as positive and current leaving (I_2 & I_4) from the node as negative.

From above diagram $-I_1 - I_2 + I_3 + I_4 + I_5 = 0$ or $I_1 + I_2 = I_3 + I_4 + I_5$

i. e., Incoming currents = Outgoing currents, Hence Kirchhoff's first law can be stated as:

In an electric circuit the sum of currents flowing towards any junction is equal to the sum of the currents flowing away from the junction

Solved Problem: Find the currents I_W, I_X, I_Y, I_Z



Solution: $I_W = -2$ **A** $I_X = -5$ **A**

$I_Y = -3$ **A** $I_Z = -8$ **A**

Kirchhoff’s Voltage Law (KVL)

Kirchhoff’s voltage law tells us how to handle voltages in an electric circuit.

Kirchhoff’s voltage law states that the algebraic sum of the voltages around any closed path is equal to zero

Or

The algebraic sum of the voltage drops is zero.

Or

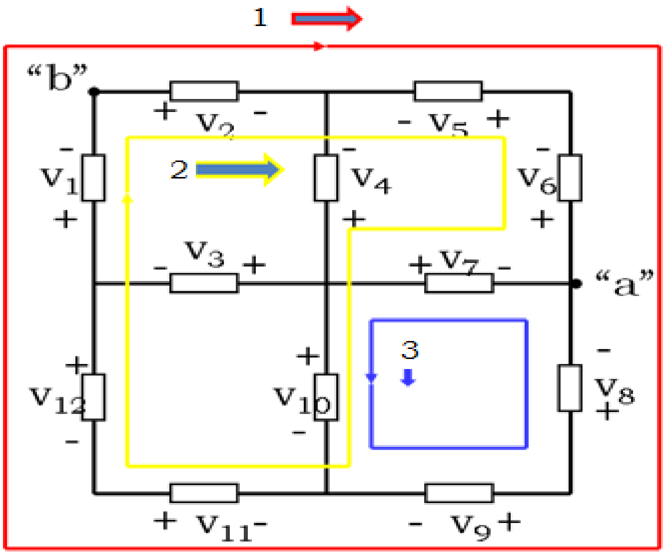
The algebraic sum of the voltage rises is zero.

Or

The algebraic sum of the voltage drops equals the algebraic sum of the voltage rises.

Solved Problem: In the following network there are a number of closed paths. Apply KVL.

Solution:



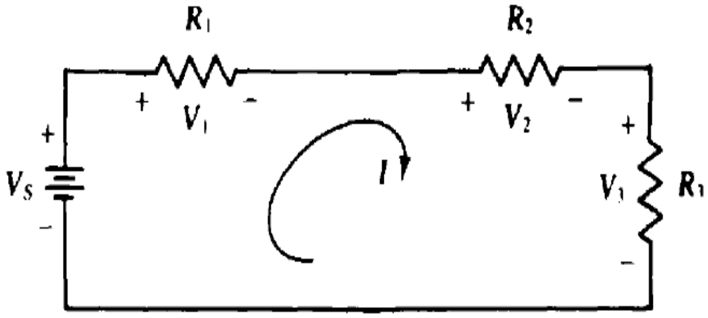
Path,1 : starting at “b” $-v_2 + v_5 + v_6 + v_8 - v_9 + v_{11} + v_{12} - v_1 = 0$

Path,2 : starting at “b” $-v_2 + v_5 + v_6 + v_7 - v_{10} + v_{11} + v_{12} - v_1 = 0$

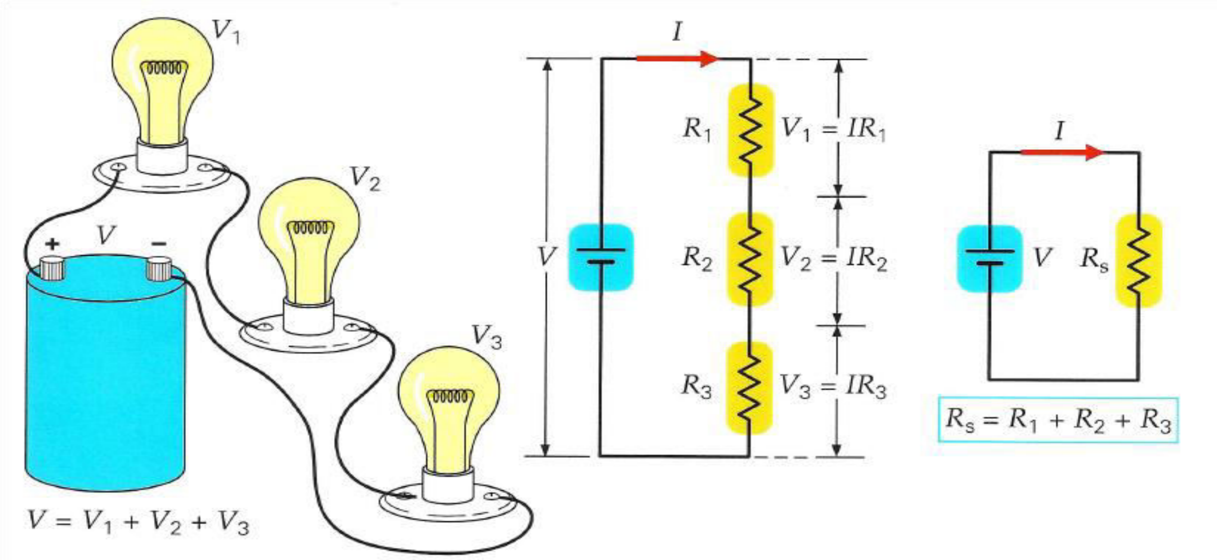
Path 3 : starting at “a” $v_7 - v_{10} + v_9 - v_8 = 0$

Series and Parallel Circuits:

In a series circuit



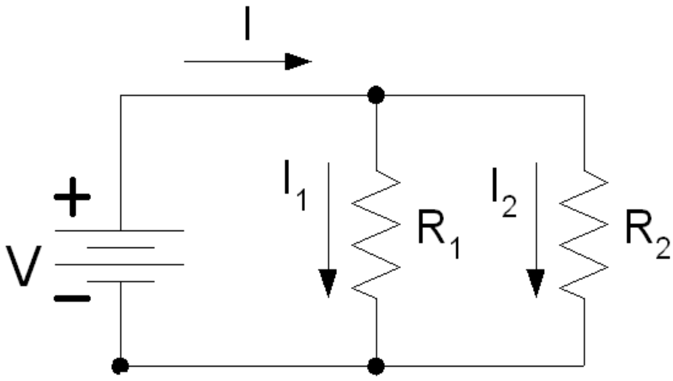
- ✓ The current I is the same in all parts of the circuit
- ✓ The sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, $V_s = V_1 + V_2 + V_3$



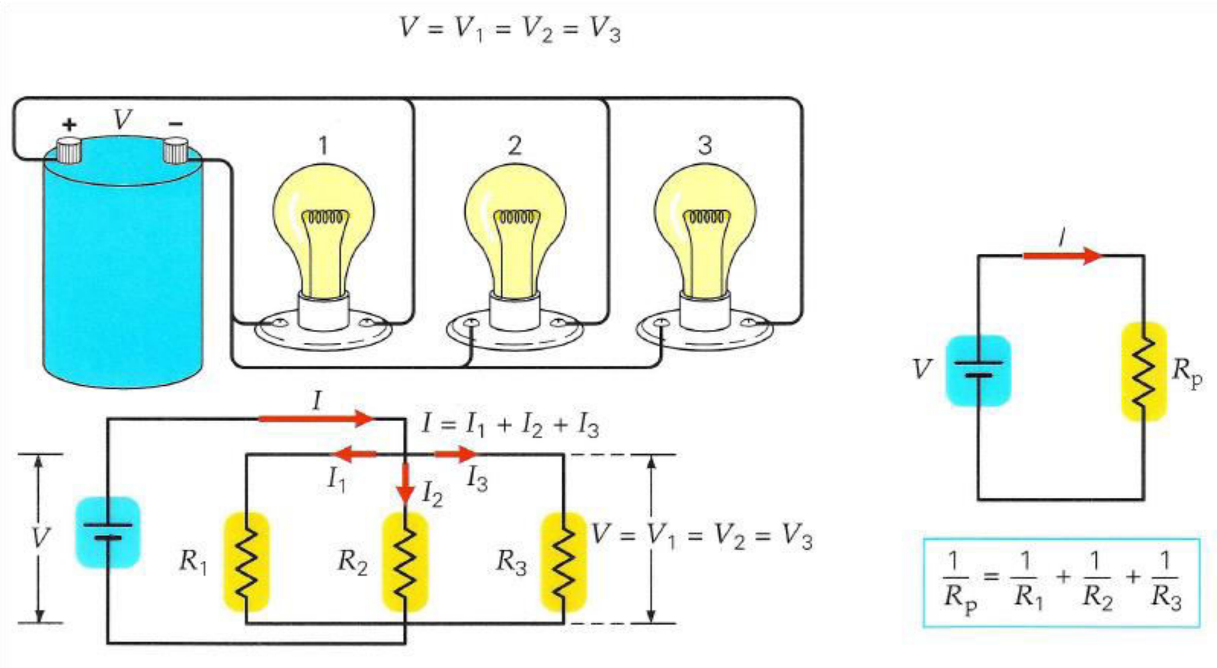
This R_s , is the total resistance of the series connected resistors. Also called as equivalent resistance, with symbol R_{eq}

For N Series Resistors $$R_{eq} = R_s = R_1 + R_2 + \cdots\cdots + R_N$$

In a parallel circuit:



- ✓ The sum of the currents I_1 and I_2 is equal to the total circuit current, I, i.e. $I = I_1 + I_2$ and
- ✓ The source Voltage, V volts, is the same across each of the resistors.



The equivalent resistance for any number of resistors in parallel (i.e. they have the same voltage across each resistor)

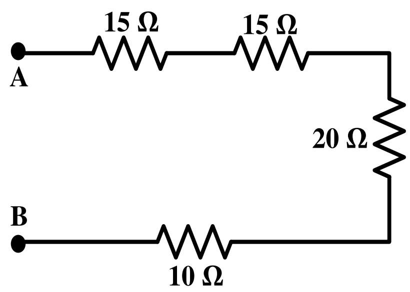
For N Parallel Resistors
$$R_{eq} = R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \ldots\ldots\ldots + \frac{1}{R_N}}$$

For Two Parallel Resistors
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Solved Problem: Find out the equivalent resistance between the nodes A &B in the given diagrams.

Solution:

In a series circuit equivalent resistance is equal to sum of the individual resistances.



$R_{eq} = R_1 + R_2 + R_3 + R_4$

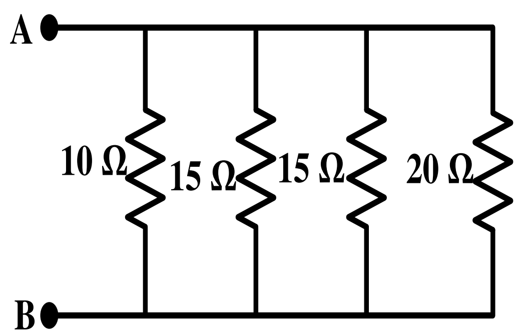
$R_{eq} = 15+15+20+10 = 60\ \Omega$

Equivalent resistance between A& B $= R_{AB} = 60\ \Omega$

Solved Problem: Find out the equivalent resistance between the nodes A &B in the given diagrams.

Solution:

In a series circuit equivalent resistance is equal to sum of the individual resistances.



$R_{eq} = R_1 + R_2 + R_3 + R_4$

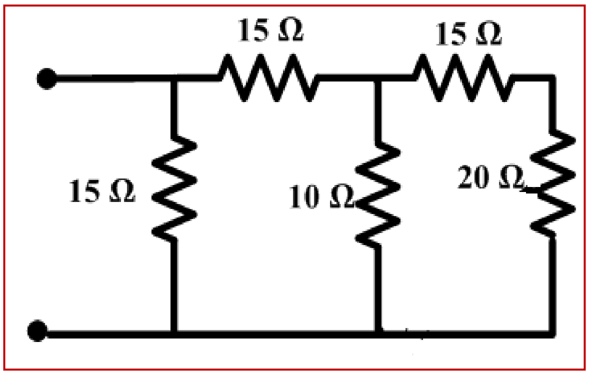
$R_{eq} = 15+15+20+10 = 60\ \Omega$

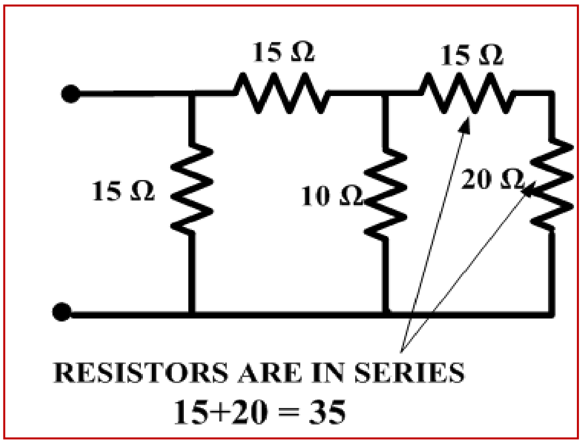
$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{15} + \frac{1}{20}$$

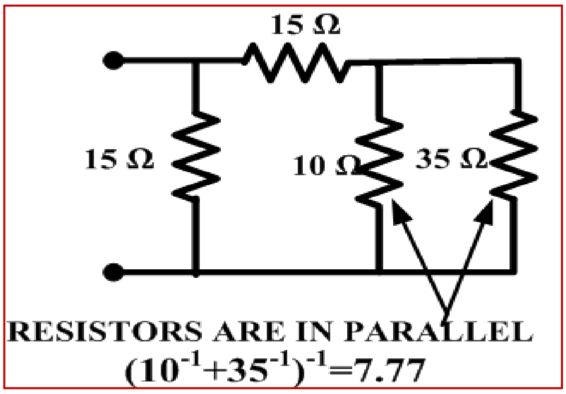
$R_{eq} = 3.52\ \Omega$

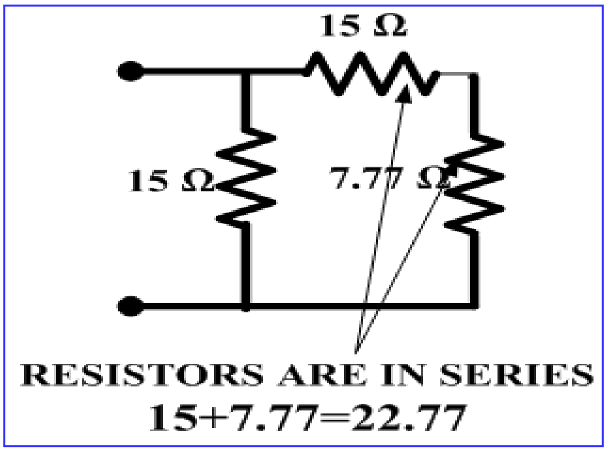
Solved Problem: Find out the equivalent resistance between the nodes

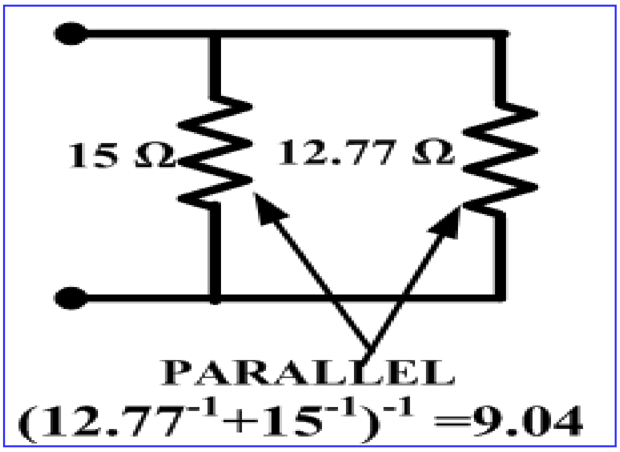
Solution:







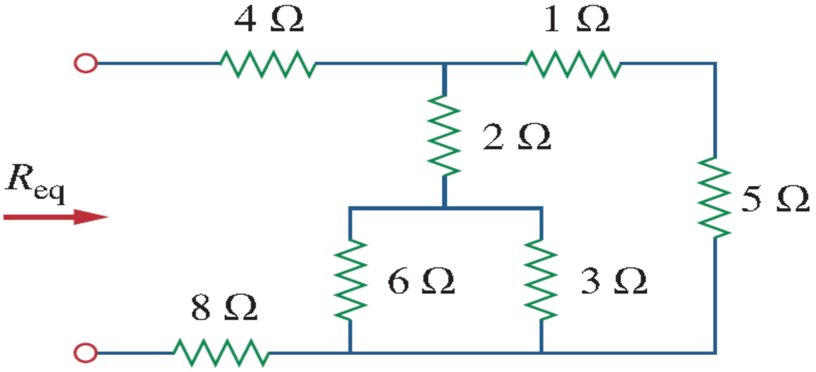


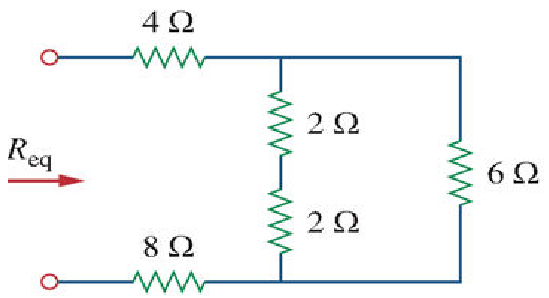


Equivalent resistance between A& B $=R_{AB}=$ 9.044 Ω

Solved Problem: Find out the equivalent resistance between the nodes

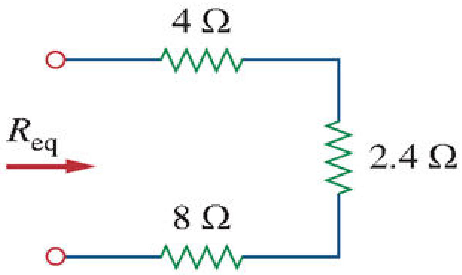
Solution:





$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$



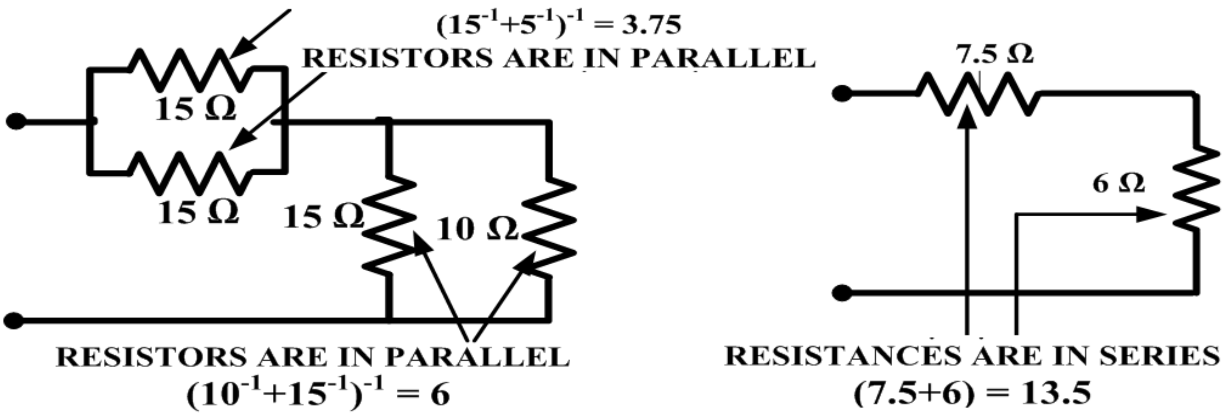
$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

$$4\,\Omega \parallel 6\,\Omega = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

$$R_{eq} = 4\,\Omega + 2.4\,\Omega + 8\,\Omega = 14.4\,\Omega$$

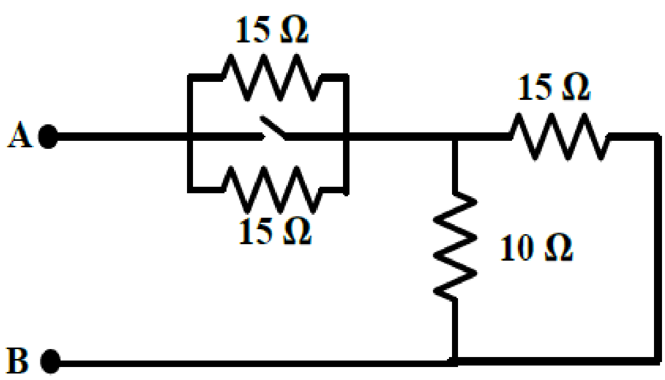
Solved Problem: Find out the equivalent resistance between the nodes

Solution:



Equivalent resistance between A& B =R_{AB}= 13.5 Ω

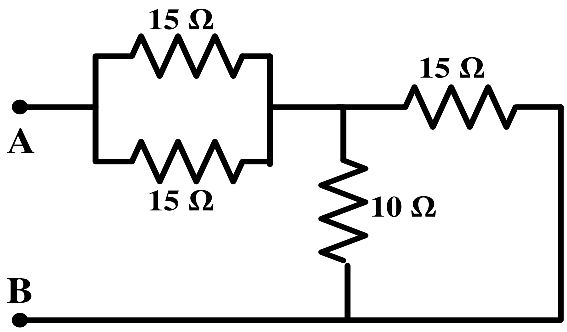
Solved Problem: Find out the equivalent resistance between the nodes A &B in the given diagrams **before and after closing switch**.



Solution:

Before closing switch.

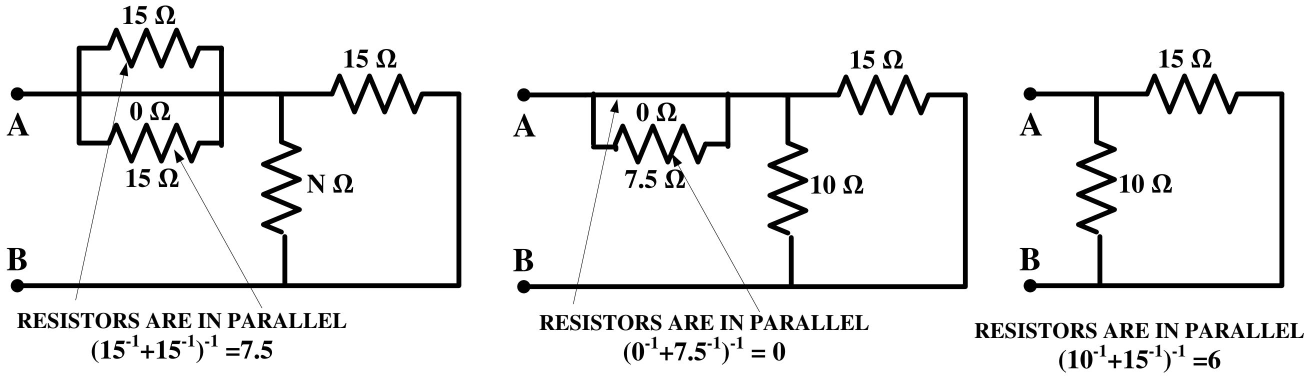
When switch is opened current flowing through switch is zero.



Equivalent resistance between A& B =R_{AB}=13.5 Ω

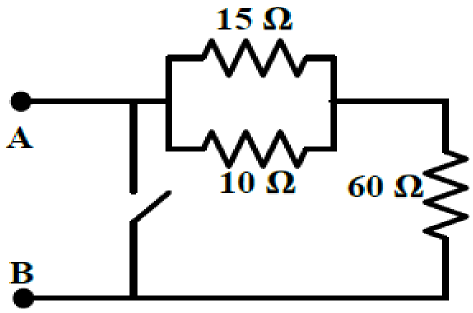
After closing switch.

When switch is closed ideal switch offers zero resistance so there is no effect of parallel resistors



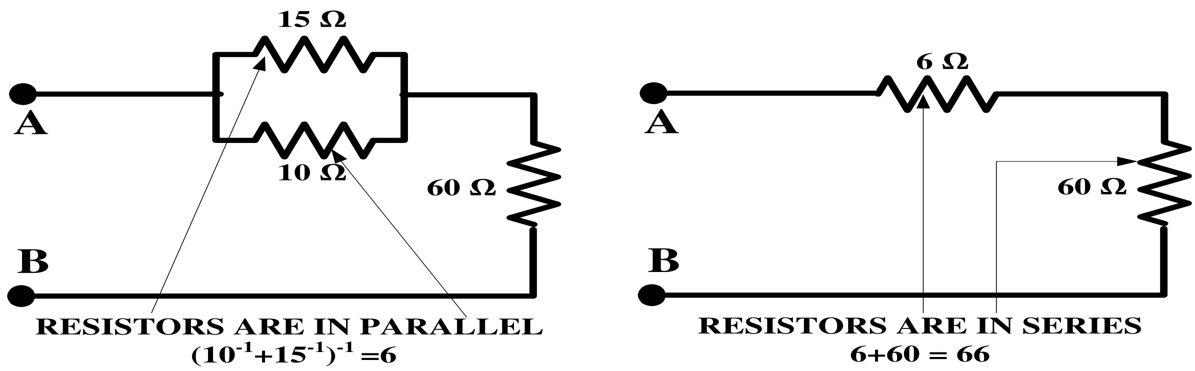
Equivalent resistance between A& B =R_{AB}=6 Ω

Solved Problem: Find out the equivalent resistance between the nodes A &B in the given diagrams **before and after closing switch**.



Solution: Before closing switch.

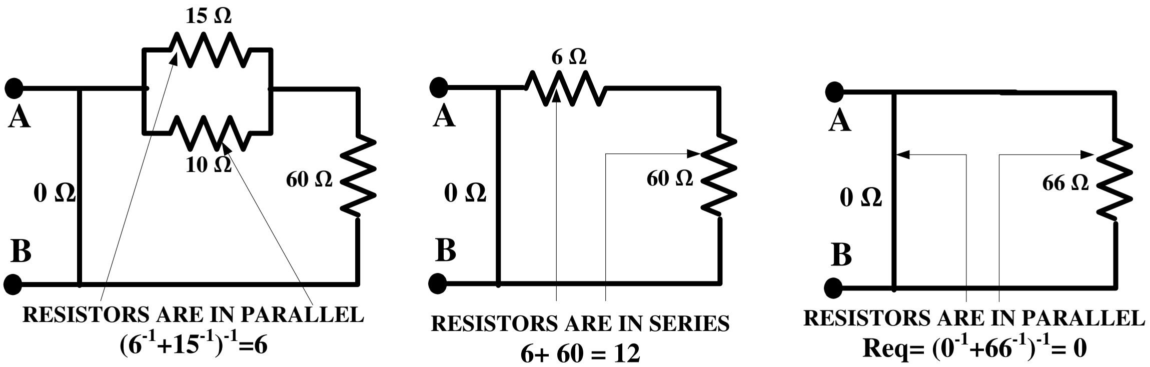
When switch is opened current flowing through switch is zero.



Equivalent resistance between A& B =R_{AB}= 66 Ω

After closing switch.

When switch is closed ideal switch offers zero resistance so there is no effect of parallel resistors



Equivalent resistance between A& B $=R_{AB}= 0\ \Omega$

<u>The voltage division Rule:</u>

In general, for any number of series resistors with a total resistance of R_s and with a voltageof V across the series combination, the voltage Vx across one of the resistors Rx , is $V_X = \dfrac{R_X}{R_s} V$

If N resistors in series , then equivalent resistance is

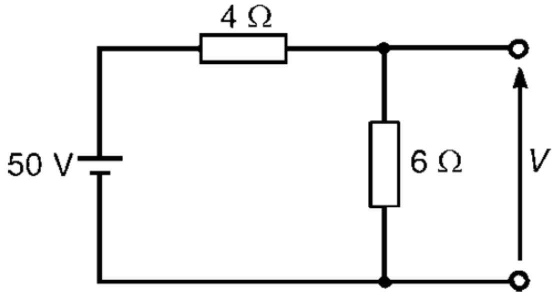
$$R_s = R_1 + R_2 + \cdots + R_N$$

$$I = \frac{V}{R_s}$$

Voltage across X^{th} branch is $V_X = \dfrac{R_X}{R_s} V$

Solved Problem : Find voltage across 6Ω

Solution:



$$V_X = \frac{R_X}{R_T} V_s$$

$$V_{6\Omega} = \frac{6}{6+4} 50 = 30V$$

Current Division Rule :

In general, for any number of parallel resistors with a total resistance of Rp and with a voltage of V across the parallel combination, the Current Ix in the x[th] resistors Rj , is: $I_X = \frac{R_p}{R_X} I$

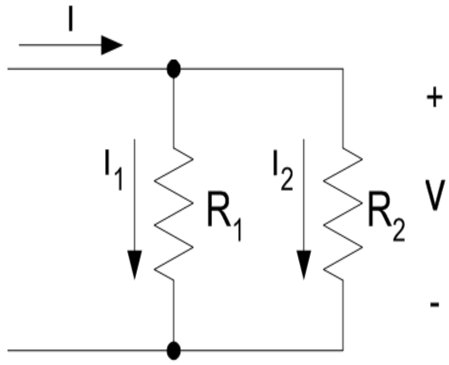
If N resistors in parallel equivalent resistance is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_N}$$

$$V = R_p I$$

Current in jth branch is $I_X = \frac{V}{R_X} = \frac{R_p}{R_X} I$

In case of two resisters in parallel

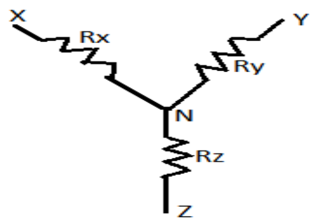


$$V = R_p I = \frac{R_1 R_2}{R_1 + R_2} I$$

$$I_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$$

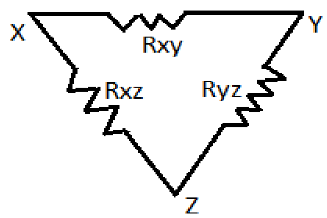
$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I$$

Star or Wie Connection(Y):



The star connection is formed with three resistors and one terminal of the three resistors is connected to the common point called star point and the remaining three terminals (open nodes) is shown in the figure.

Delta Connection (Δ):



The Delta connection is formed with three resistors in a closed loop forming the three nodes

Star – Delta Transformation:

The star to delta and delta to star transformation is possible based on the assumption that the equivalent resistance between any two terminals of the connections before and after the transformation must be same.

i.e, $R_{X\&Y \text{ in satr (Y)}} = R_{X\&Y \text{ in delta}(\Delta)}$

$R_{Y\&Z \text{ in satr (Y)}} = R_{Y\&Z \text{ in delta}(\Delta)}$

$R_{X\&Z \text{ in satr (Y)}} = R_{X\&Z \text{ in delta}(\Delta)}$

Therefore in Star connection,

$R_{X\&Y\text{ in satr (Y)}} = R_x + R_y$ ... (1)

$R_{Y\&Z\text{ in satr (Y)}} = R_y + R_z$ ... (2)

$R_{X\&Z\text{ in satr (Y)}} = R_x + R_z$ ... (3)

Similarly in Delta connection,

$$R_{X\&Y\text{ in delta }(\Delta)} = \frac{R_{xy}(R_{yz}+R_{xz})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (4)}$$

$$R_{Y\&Z\text{ in delta }(\Delta)} = \frac{R_{yz}(R_{xy}+R_{xz})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (5)}$$

$$R_{X\&Z\text{ in delta }(\Delta)} = \frac{R_{xz}(R_{yz}+R_{xy})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (6)}$$

Based on the assumption equate equations (1), (4); (2),(5) and (3),(6)

$$R_x + R_y = \frac{R_{xy}(R_{yz}+R_{xz})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (7)}$$

$$R_y + R_z = \frac{R_{yz}(R_{xy}+R_{xz})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{...(8)}$$

$$R_x + R_z = \frac{R_{xz}(R_{yz}+R_{xy})}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (9)}$$

Delta to star transformation:

Simplify the above three equations, the formulae for star connected resistances(R_x, R_y and R_z) are obtained in terms of the given delta connected resistances (R_{xy}, R_{yz} and R_{xz})

Subtract Equation (7) and equation (8) = equation (10)

$$R_x - R_z = \frac{R_{xy}R_{xz} - R_{yz}R_{xz}}{R_{xy}+R_{yz}+R_{xz}} \quad \text{...(10)}$$

On adding equations (10) and equation (9) = equation (11)

$$2R_x = \frac{R_{xy}R_{xz} - R_{yz}R_{xz} + R_{xy}R_{xz} + R_{yz}R_{xz}}{R_{xy}+R_{yz}+R_{xz}} = \frac{2R_{xy}R_{xz}}{R_{xy}+R_{yz}+R_{xz}}$$

Therefore

$$R_x = \frac{R_{xy}R_{xz}}{R_{xy}+R_{yz}+R_{xz}} \quad \text{...(11)} \qquad R_y = \frac{R_{xy}R_{yz}}{R_{xy}+R_{yz}+R_{xz}} \quad \text{...(12)} \qquad R_z = \frac{R_{xz}R_{yz}}{R_{xy}+R_{yz}+R_{xz}} \quad \text{... (13)}$$

Thus equations 11, 12 and 13 are the formulas for converting the given delta connected resistors to star connected values.

Star to Delta transformation:

Multiply equations 11,12 and equations 12,13 and also equations 11,13.

$$R_x R_y + R_y R_z + R_x R_z = \frac{R_{xy}^2 R_{yz} R_{xz} + R_{xy} R_{yz}^2 R_{xz} + R_{xy} R_{yz} R_{xz}^2}{\left(R_{xy} + R_{yz} + R_{xz}\right)^2} = \frac{R_{xy} R_{yz} R_{xz} (R_{xy} + R_{yz} + R_{xz})}{\left(R_{xy} + R_{yz} + R_{xz}\right)^2}$$

$$= \frac{R_{xy} R_{yz} R_{xz}}{\left(R_{xy} + R_{yz} + R_{xz}\right)}$$

Also divide the above equation with R_z on both sides from eq.13

$$\frac{R_x R_y + R_y R_z + R_x R_z}{R_z} = \frac{R_{xy} R_{yz} R_{xz}}{\left(R_{xy} + R_{yz} + R_{xz}\right)} * \frac{\left(R_{xy} + R_{yz} + R_{xz}\right)}{R_{yz} R_{xz}} = R_{xy}$$

$R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$ ....(14) Similarly $R_{yz} = R_y + R_z + \frac{R_y R_z}{R_x}$ ......(15) and

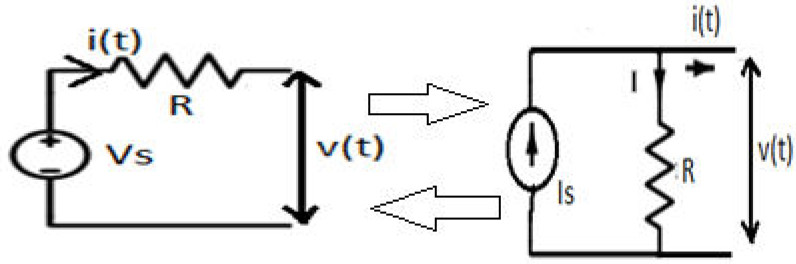
$R_{xz} = R_x + R_z + \frac{R_x R_z}{R_y}$ ....(16) .

Source transformation:

Using this transformation the voltage source can be transformed to current source and the current source to voltage source

This technique is applicable only for the practical sources

Let the voltage source shown in the below figure is to be converted to current source



KVL to the loop of voltage source is

$$v(t) = V_S - i(t)R$$

$$V_S = v(t) + i(t)\ R \quad ......(1)$$

Similarly $i(t) = I_S - \frac{v(t)}{R}$

$$I_S = i(t) + \frac{v(t)}{R} \quad ......(2)$$

Multiply the eq.(2) on both sides with R, then

$$I_S R = i(t)\ R + \frac{v(t)}{R} R \Rightarrow I_S R = v(t) + i(t)\ R \quad .....(3)$$

Comparing the equations (1) and (3), the RHS sides are equal therefore, LHS sides should be same

i.e, $V_s = I_s R$

Thus, the rules for transformation are

1. Place the internal resistance in series for Voltage source, and in parallel for current source with same resistance value.

2. Voltage magnitude after transformation from current source is $V_s = I_s R$ volt and Current magnitude after transformation from voltage source is $I_s = V_s / R$

Summary: Important Formulas in unit – 1

Basic terms

$$I = \frac{Q}{t} \quad or \quad i = \frac{dq}{dt}$$

$$v_{ab} = \frac{dw}{dq} \quad or \quad V = \frac{W}{Q}$$

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

$$W = Pt$$

V-I Relations in R,L and C

$$R = \frac{V}{I}$$

$$V = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_0^t V dt + i(0)$$

$$p = Li\frac{di}{dt}$$

$$W = \frac{1}{2}Li^2$$

$$i = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_0^t i dt + v(0)$$

$$p = Cv\frac{dv}{dt}$$

$$W = \frac{1}{2}Cv^2$$

Series and Parallel

$$R_{eq} = R_s = R_1 + R_2 + \cdots\cdots + R_N$$

$$R_{eq} = R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \cdots\cdots + \frac{1}{R_N}}$$

Voltage and current division rules

$$V_X = \frac{R_X}{R_s} V$$

$$I_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I$$

Star to delta And Delta to Star Transformation

Delta to star

$$R_x = \frac{R_{xy} R_{xz}}{R_{xy} + R_{yz} + R_{xz}}$$

$$R_y = \frac{R_{xy} R_{yz}}{R_{xy} + R_{yz} + R_{xz}}$$

$$R_z = \frac{R_{xz} R_{yz}}{R_{xy} + R_{yz} + R_{xz}}$$

Star to delta

$$R_{xy} = R_x + R_y + \frac{R_x R_y}{R_z}$$

$$R_{yz} = R_y + R_z + \frac{R_y R_z}{R_x}$$

$$R_{xz} = R_x + R_z + \frac{R_x R_z}{R_y}$$